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The Force Distribution Exerted by Surface Waves on Piles

by

J. R. Morison

Berkeley, California
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Acknowledgment

The author appreciates the help given by D. M. Nelson, W. E. Pippin, M. M. Lincoln, and M. E. Conklin. The project was supervised by Professor J. W. Johnson.
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Abstract: Experimental data for the force distribution on three model cylindrical piles for three wave conditions are presented. These results are compared to the previously published theory.\(^{(1)}\)

Introduction: The purpose of this report is to present experimental data on the force distribution exerted by surface waves on piles. From these data the coefficients of drag and mass that appear in the equation\(^{(1)}\) for the force were obtained. The experimental results were compared to the calculated force distribution. In order to simplify the presentation, the force (lbs) was divided by the projected area (ft\(^2\)) of a segment of the pile to give a force intensity (lbs/ft\(^2\)). The measurement of force was made on a one-inch high segment of several model piles of various diameters. The results of these studies are that the experimentally determined coefficient of mass shows good agreement with the theoretical value of 2.0\(^{(7,8)}\), and relatively good agreement with the values in previously presented experiments\(^{(1,2,3,4,5,6)}\). The results also show that the experimentally determined coefficient of drag is in relatively good agreement with the value 1.6 as found in previous experiments\(^{(1,2,3,4,6)}\). The measurements of the force intensity distribution showed good agreement with the calculated distribution using the previously mentioned values of the coefficients in the equation for the force.

Experimental Set-up: The experiments were conducted in the 1 ft by 3 ft by 60 ft wave channel in the Fluid Mechanics Laboratory of the University of California, Berkeley. A summary of the pile sizes and wave conditions is given in Tables I and II. The force intensity was obtained by measuring the total horizontal force on a one-inch segment of the pile and dividing by the projected area of this segment. This segment could be placed over a range of elevations above the bottom of the channel. The apparatus resembled a pendulum restrained at the top, pivoted at approximately the middle, with the one-inch pile segment fastened at the bottom (See Figure 1). The displacement of the pile segment was converted to a force by means of a conversion factor obtained by calibration. In order to obtain a flow pattern similar to a continuous pile and to reduce the tare on the pendulum rod, a cylindrical shroud representing a pile was placed between the one-inch segment and the pivot (which was always above the wave surface). By the use of a frame, a dummy pile section was held below the one-inch segment to represent the lower portion of a continuous pile. The tare of the system without the segment was a very small percentage of the force on the segment. The natural frequency of the system was relatively near the frequency of the uniform, periodic wave trains. This caused considerable, unavoidable trouble which the one-inch segment was near the surface of the waves; especially when the waves were very steep and when the waves were in relatively shallow water.

\(^{(1)}\) Numbers in ( ) are reference numbers.
Force Equation (1): The total horizontal force on the one-inch segment of the pile is given by the expression

\[ dF = \frac{1}{2} \rho C_D u^2 D \, dS + \rho C_M \frac{\partial u}{\partial t} \frac{\pi D^2}{4} \, dS \]  

(1)

where

- \( dF = \text{total force on pile segment, } dS, (\text{lbs}) \)
- \( \rho = \text{density of water, } (\text{lbs} \times \text{sec}^2/\text{ft}^4) \)
- \( C_D = \text{coefficient of drag} \)
- \( C_M = \text{coefficient of mass} \)
- \( u = \text{horizontal particle velocity (ft/sec)} = \frac{\pi H}{T} \frac{\cosh \frac{2\pi S}{L}}{\sinh \frac{2\pi d}{L}} \cos \theta \)
- \( \frac{\partial u}{\partial t} = \text{horizontal particle acceleration (ft/sec}^2) = \frac{2\pi^2 H}{T^2} \frac{\cosh \frac{2\pi S}{L}}{\sinh \frac{2\pi d}{L}} \sin \theta \)

- \( D = \text{pile diameter (ft)} \)
- \( dS = \text{segment of pile (ft)} \)
- \( H = \text{wave height - (ft)} \)
- \( T = \text{wave period - (sec)} \)
- \( L = \text{wave length - (ft)} \)
- \( d = \text{still-water depth (ft)} \)
- \( S = \text{elevation of the section } dS \text{ above the bottom (ft)} \)
- \( \theta = \text{angular position of particle in its orbit} \)
- \( \text{measure counterclockwise (degrees)} \)

The first term of Equation (1) is the drag force and the second term is the inertia force. The coefficient of mass, \( C_M \), includes the virtual mass.

The force intensity is given by the expression

\[ P = \frac{dF}{dS} \]  

(2)

Substituting Equation (1) into Equation (2), together with the expressions for \( u \) and \( \frac{\partial u}{\partial t} \), the force intensity becomes

\[ P = \frac{1}{2} \rho \frac{\pi^2 H^2}{T^2} \left[ C_D K^2 \cos^2 \theta + C_M K \left( \frac{\pi D}{H} \right) \sin \theta \right] \]  

(3)

where

\[ K = \frac{\cosh \frac{2\pi S}{L}}{\sinh \frac{2\pi d}{L}} \]

The angular position, corresponding to the wave crest, the still-water level, the trough and the following still-water level are 0°, 90°, 180°, and 270°, respectively. In order to obtain the position of the maximum force intensity relative to the wave position, Equation (3) is differentiated with respect to \( \theta \), set equal to zero, and solved for \( \theta \). The result, called the phase angle of the maximum force intensity, is measured from the crest position, and is denoted by \( \theta \) (max).
The position of the minimum force intensity is obtained in the same manner and is denoted by $\beta$ (min).

$$\beta_{\text{min}} = \beta_{\text{max}} + 180^\circ$$

Equations 4 and 5 do not hold for $dS$ above the trough elevation so that the maximum and minimum force intensity for these elevations can only be obtained from a plot of Equation 5 for the values of $\Theta$ for which the surface elevation $(S_c)$ is greater than the elevation $(S)$ of the segment $(dS)$. The position and magnitude of the maximum force intensity relative to the wave crest then can be obtained from this graph.

The notation $P(\Theta)$, $P(90^\circ)$, etc., indicates that Equation (3) has been solved for $\Theta = 0^\circ$, $90^\circ$, etc., respectively. The notation $P_{\text{max}}$ and $P_{\text{min}}$ indicates that Equation (3) has been solved for $\Theta = \Theta_{\text{max}}$ and $\Theta_{\text{min}}$, respectively. Since the force intensity is also a function of the elevation, $S$, the greatest $P_{\text{max}}$ would be obtained for the larger values of $K_1$; that is, for the large values of $S$, which is when the crest impinges on the pile. Conversely, the largest of the $P_{\text{min}}$ values occurs when the trough of the wave acts on the pile. However, this is only true if $C_D$ and $C_M$ are constants over the length of the pile. In this study it was found that for all practical purposes these coefficients were constants. However, if these two coefficients were not constant over the length of the pile, their variations with $S$ and $\Theta$ must be included when obtaining the greatest $P_{\text{max}}$ and $P_{\text{min}}$.

Results and Discussion: The results of the experimental study of force intensity distribution for three different size piles for two wave conditions are presented in Tables I and II. Two values of the force intensity are given in Table I. The first value is for a solid one-inch pile segment and the second value is for a hollow one-inch pile segment. These sections were used to show that the mass of the moving segment did not affect the results. The motion of the segment was small, being of the order of $1/32$ of an inch maximum. The effect of the two different masses was to change the natural frequency of the system, and it was found that the hollow segment proved more suitable for the range of wave frequencies possible in the wave channel. Hence, it was used for the remainder of the experiments. Resonant vibration of the recording system occurred when measurements were taken near the surface of the water causing poor results which are not presented.

At the top of each table there is a summary of the average wave condition together with the maximum percentage deviation of any measurement from that average value. All deviations in the wave conditions were less than $10\%$, and about half of them were less than $5\%$. The deviation in the measurement of the force intensity was within $10\%$ of the maximum measured force intensity as shown in Table I, where the same results were obtained from two different experimental set-ups.

In Table I the wave in deep water was of moderate steepness ($H/L = 0.039$). The inertia force was predominant with the drag force becoming more important for the small pile, and near the wave surface. The inertia force had
a larger relative effect under the crest than under the trough. The maximum and minimum force intensities are approximately equal for this wave condition so that the time history of the forces are nearly symmetrical about the maximum ordinate and zero abscissa (see also Figure 6).

In Table II the wave in deep water was very steep ($H/L = 0.091$). This wave was the same length as the wave condition of Table I, but the height was greater. The data show the drag forces to have been more predominant than for the less steep wave (Table I). Otherwise the trends of the data were about the same.

Thus far, the data indicate that the drag force is predominant for small piles, for steep waves, for shallow-water waves, for the segment of piling near the surface, and for condition when the wave crest passes the pile. In other words, the drag force is important in regions of high relative velocity, i.e. turbulence. The inertia force is predominant for relatively large piles, for waves of relatively small steepness, for deep-water waves, for segments of the pile well below the wave surface and when the wave position was such that the pile was at the still-water level. The inertia force is important in regions of high acceleration where large masses are involved. It must be remembered that this report deals only with horizontal forces.

The data presented in Tables I and II in the columns $P(0)$, $P(90)$, $P(180)$, $P(270)$, together with the wave dimensions and pile geometry, were substituted into Equation 3 from which the values of $C_D$ and $C_M$ were computed. The average $C_D$ and $C_M$ obtained from this study are shown in Table III. The value of $C_D = 2.03$ compares roughly to the previously reported value of $1.6(1)$ (25% variation). The value of $C_M = 1.96$ compares extremely well with the theoretical value of $2.0(7.8)$ and roughly with the previously reported value of $1.6(1)$ (25% variation).

Figures 2 through 5 which show the drag component and the inertia component of force intensity are presented in dimensionless form in order to show the distribution, relative magnitude, and relative deviation between measured and computed force intensity. The coefficients used in the computed force intensity were $C_D = 1.6$ and $C_M = 2.0$. The force intensity was made dimensionless by dividing by the greatest computed maximum force intensity possible for a given pile and a given wave condition. Figures 2 and 4 are for waves of relatively small steepness in deep water (Table I); and Figures 3 and 5 are for a very steep wave in deep water (Table II). Figures 2 and 3 show the drag components $P(0)$ and $P(180)$ of the force intensity. The figures show that the drag force is more predominant for the smallest pile, in shallow water, and for steep waves. The agreement between the measurements and the calculations is fair considering that most of the measurements are less than 20% for the maximum amplitude, with a deviation of generally less than 5% of that maximum amplitude. Figures 3 and 4 show the inertia components $P(90)$ and $P(270)$ of the force intensity. The inertia force is shown to be more predominant for the larger piles, for deep water and for relatively low waves. The agreement between measured and calculated force intensity is generally within 10% of the maximum amplitude. Theoretically, the second order effects of the force intensity would show that the force intensity under the crest is greater than the force intensity under the trough, however this effect was not distinguishable in this study. The force intensity computed by Equation 3, for the wave condition of Table I and Figures 2 and 4, was plotted in
Figure 6. This example shows the force intensity at the wave surface, hence at the elevation $S_h$, which varies with $\theta$. The curves for the different pile sizes show the shift of the phase angle, and hence the relative importance of the drag and inertia components of the horizontal force. The curves also show the difference between the force intensity at the crest (maximum $S_h$) and at the trough (minimum $S_h$). Unfortunately no suitable measurements could be obtained at the wave surface.

Conclusions: The good agreement between the measurements and the calculated force intensity indicate the following:

(1) The values of the coefficients for the force equation of $C_D = 1.6$ and $C_M = 2.0$ were suitable for the calculation of the force intensity in this model study.

(2) The drag component of the horizontal force exerted by surface waves on cylindrical piles is predominant for small piles, for steep waves, for shallow water, for segments of the pile near the surface and for the condition when the wave crest passes the pile.

(3) The inertia component of the horizontal force exerted by surface waves on cylindrical piles is predominant for large piles, for waves of small steepness, for deep water, for segments of the pile well below the wave surface and when the wave position is such that the pile is at the still-water level.

References


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<td>$C_M = 1.96 \pm 0.25 \quad (1.15 - 2.83)$</td>
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| Experimentally Determined Coefficient of Drag |
| Average value, average deviation and range |
| $C_D = 2.03 \pm 0.40 \quad (0.98 - 3.50)$ |
### Table I

**MEASURED FORCE DENSITY ON MODEL PILES**

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<th>S (ft)</th>
<th>P104 (104 lb/ft²)</th>
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*Top number is experiment with solid cylinder

Bottom number is experiment with hollow cylinder

### Table II

**MEASURED FORCE DENSITY ON MODEL PILES**

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